

## Riešenie príkladov z 10. kapitoly

**Cvičenie 10.1.** Aká je hodnota Boolovej premennej, ktorá je určená podmienkou

- (a)  $x \cdot 1 = 0$  ( $x = 0$ ),
- (b)  $x + x = 0$  (pre  $x = 0$ ),
- (c)  $x \cdot 1 = x$  (pre každé  $x$ ),
- (d)  $x + \bar{x} = 1$  (pre každé  $x$ ),
- (e)  $x \cdot \bar{x} = 0$  (pre každé  $x$ ).

**Cvičenie 10.2.** Zostrojte tabuľku funkčných hodnôt Boolovej funkcie

(a)  $f(x, y, z) = \bar{x}y$ ,

#	$x$	$y$	$z$	$f$
1	0	0	0	0
2	0	0	1	0
3	0	1	0	1
4	0	1	1	1
5	1	0	0	0
6	1	0	1	0
7	1	1	0	0
8	1	1	1	0

(b)  $f(x, y, z) = x + yz$ ,

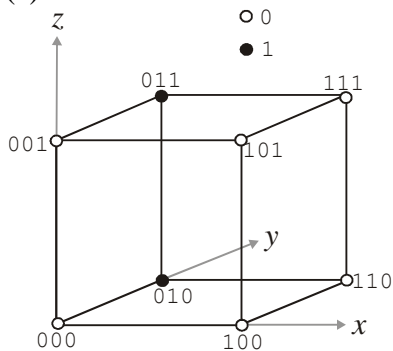
#	$x$	$y$	$z$	$f$
1	0	0	0	0
2	0	0	1	0
3	0	1	0	0
4	0	1	1	1
5	1	0	0	1
6	1	0	1	1
7	1	1	0	1
8	1	1	1	1

(c)  $f(x, y, z) = x\bar{y} + \bar{x}y\bar{z}$ ,

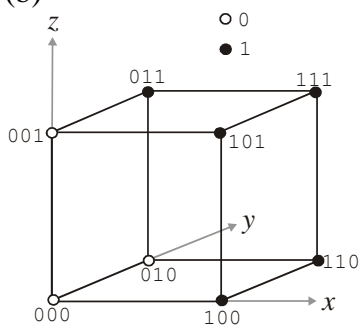
#	$x$	$y$	$z$	$x\bar{y}$	$\bar{x}y\bar{z}$	$f$
1	0	0	0	0	1	1
2	0	0	1	0	0	0
3	0	1	0	0	0	0
4	0	1	1	0	0	0
5	1	0	0	1	0	1
6	1	0	1	1	0	1
7	1	1	0	0	0	0
8	1	1	1	0	0	0

**Cvičenie 10.3.** Znázornite Boolove funkcie  $f(x, y, z)$  z cvičenia 9.2 na 3-rozmernej kocke tak, že hodnoty 1 (0) budú reprezentované na kocke čiernym (bielym) bodom.

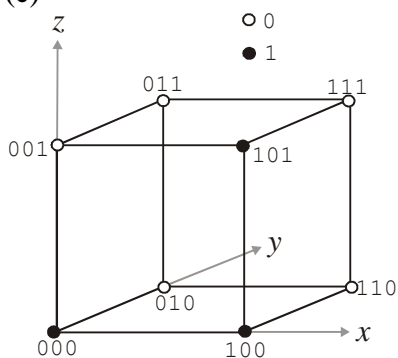
(a)



(b)



(c)



**Cvičenie 10.4.** Pre ktoré hodnoty  $x$  a  $y$  platí  $xy = x + y$ .

(a) Prípád  $0 = 0$ , potom  $x = y = 0$ ,

(b) Prípád  $1 = 1$ , potom  $x = y = 1$ .

**Cvičenie 10.5.** Zostrojte tabuľku všetkých možných binárných Boolových funkcií z tabuľky 1.8 a identifikujte v nej Boolove binárne operácie pomocou súčinu, súčtu a negácie.

$f_1 = 0$	$f_2 = xy$	$f_3 = x\bar{y}$	$f_4 = x\bar{y} + xy$
$f_5 = \bar{x}y$	$f_6 = \bar{x}y + xy$	$f_7 = \bar{x}y + x\bar{y}$	$f_8 = \bar{x}y + x\bar{y} + xy$
$f_9 = \bar{x}\bar{y}$	$f_{10} = \bar{x}\bar{y} + xy$	$f_{11} = \bar{x}\bar{y} + x\bar{y}$	$f_{12} = \bar{x}\bar{y} + x\bar{y} + xy$
$f_{13} = \bar{x}\bar{y} + \bar{x}y$	$f_{14} = \bar{x}\bar{y} + \bar{x}y + xy$	$f_{15} = \bar{x}\bar{y} + \bar{x}y + x\bar{y}$	$f_{16} = 1$

**Cvičenie 10.6.** Riešte nasledujúce rovnice s exkluzívnou disjunkciou

- (a)  $x \oplus \mathbf{0} = x$ ,
- (b)  $x \oplus \mathbf{1} = \bar{x}$ ,
- (c)  $x \oplus x = \mathbf{0}$ ,
- (d)  $x \oplus \bar{x} = \mathbf{1}$ .

**Cvičenie 10.7.** Dokážte, že platia rovnosti

(a)  $x \oplus y = (x + y)(\overline{xy})$ ,

$$x \oplus y = (x + y)(\overline{xy}) = (x + y)(\bar{x} + \bar{y}) = \cancel{x\bar{x}} + x\bar{y} + \bar{x}y + \cancel{y\bar{y}}$$

$$= \begin{cases} 0 & (x = y) \\ 1 & (x \neq y) \end{cases}$$

(b)  $x \oplus y = \bar{x}y + x\bar{y}$ ,

Dokáže sa pomocou predošlého riešenia (a).

**Cvičenie 10.8.** Zostrojte duálne výrazy k týmto Boolovým funkciám

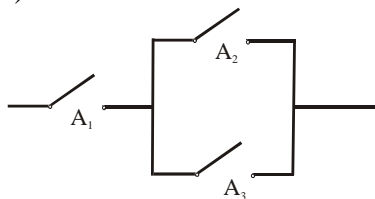
- (a)  $x + y$ ,  $(\bar{x}\bar{y})$
- (b)  $\bar{x}\bar{y}$ ,  $(x + y)$
- (c)  $xyz + \bar{x}\bar{y}\bar{z}$ ,  $((\bar{x}\bar{y}\bar{z})(xyz))$ .

**Cvičenie 10.9.** Zostrojte Boolovu funkciu  $f(x, y, z)$  vo forme sumy produktov klauzúl k premenným  $x, y$  a  $z$ , ktorá má hodnotu **1** vtedy a len vtedy, ak

- (a)  $x = y = \mathbf{0}, z = \mathbf{1}$ ,  $(\bar{x}\bar{y}z)$
- (b)  $x = \mathbf{0}, y = \mathbf{1}, z = \mathbf{0}$ ,  $(\bar{x}y\bar{z})$
- (c)  $y = z = \mathbf{1}$ ,  $(xyz + \bar{x}yz)$ .

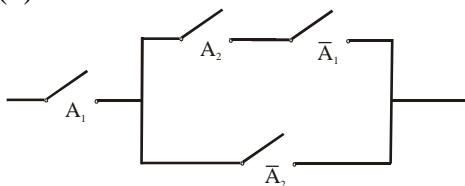
**Cvičenie 10.10.** Zostrojte spínacie funkcie pre spínacie obvody

(a)



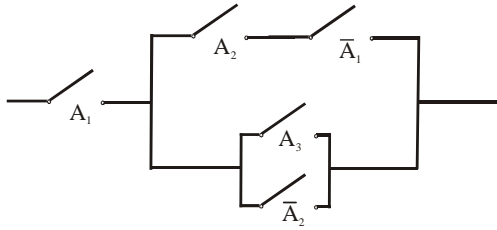
$$x_1(x_2 + x_3)$$

(b)



$$x_1((x_2\bar{x}_1) + \bar{x}_2)$$

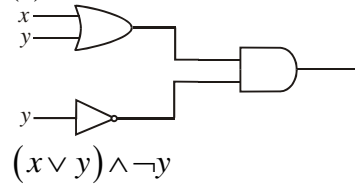
(c)



$$x_1(x_2\bar{x}_1 + (x_3 + \bar{x}_2))$$

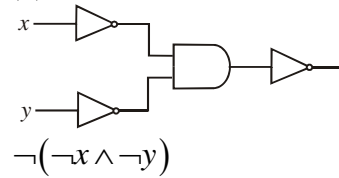
**Cvičenie 10.11.** Zostrojte tabuľku výstupov logických obvodov

(a)



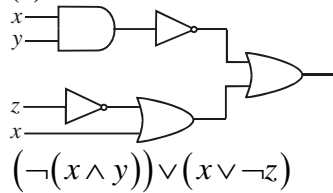
$$(x \vee y) \wedge \neg y$$

(b)



$$\neg(\neg x \wedge \neg y)$$

(c)



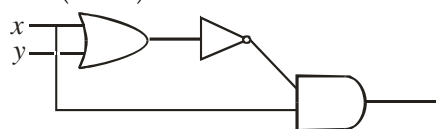
$$\neg(x \wedge y) \vee (x \vee \neg z)$$

**Cvičenie 10.12.** Zostrojte logické obvody, ktoré simulujú Boolove funkcie

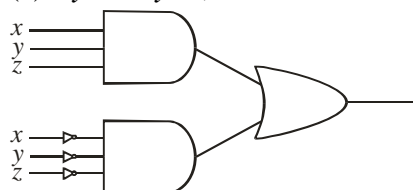
(a)  $\bar{x} + y$ ,



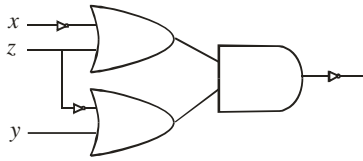
(b)  $(\overline{x+y})x$ ,



(c)  $xyz + \bar{x}\bar{y}\bar{x}$ ,

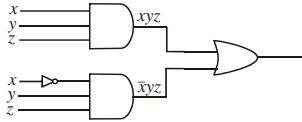


(d)  $\overline{(\bar{x} + z)(y + \bar{z})}$ .



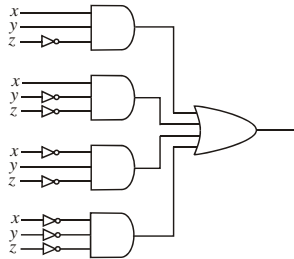
**Cvičenie 10.13.** Zjednodušte logické obvody

(a)



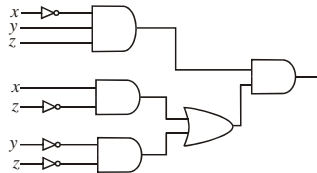
$$xyz + \bar{x}yz = (x + \bar{x})yz = yz$$

(b)



$$xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z} = x(y + \bar{y})\bar{z} + \bar{x}(y + \bar{y})\bar{z} = x\bar{z} + \bar{x}\bar{z} = (x + \bar{x})\bar{z} = \bar{z}$$

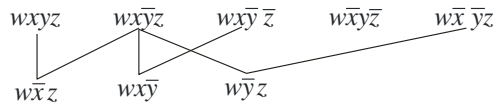
(c)



$$(\bar{x}yz)(x\bar{z} + y\bar{z}) = \bar{x}yzx\bar{z} + \bar{x}yzy\bar{z} = 0 + 0 = 0$$

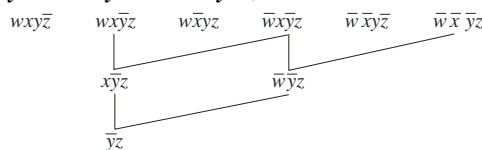
**Cvičenie 10.14.** Pomocou Quinovej a McCluskeyho metódy nájdite optimálne výrazy k Boolovým funkciám

(a)  $wxyz + wx\bar{y}z + wx\bar{y}\bar{z} + w\bar{x}y\bar{z} + w\bar{x}\bar{y}z$ ,



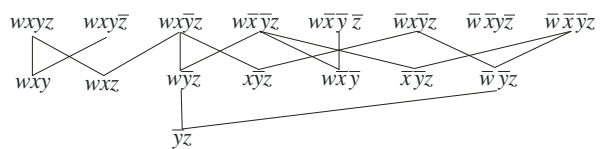
$$f = w\bar{x}z + wx\bar{y} + w\bar{y}z + w\bar{x}y\bar{z}$$

(b)  $wxy\bar{z} + wx\bar{y}z + w\bar{x}yz + w\bar{x}\bar{y}z + w\bar{x}y\bar{z} + w\bar{x}\bar{y}z$ ,



$$f = \bar{y}z + wxy\bar{z} + w\bar{x}yz + w\bar{x}\bar{y}z$$

(c)  $wxyz + wx\bar{y}z + wx\bar{y}\bar{z} + w\bar{x}y\bar{z} + w\bar{x}\bar{y}\bar{z} + w\bar{x}y\bar{z} + w\bar{x}\bar{y}z + w\bar{x}\bar{y}z$ .



$$f = wxy + \bar{y}z + w\bar{x}\bar{y} + \bar{w}\bar{x}y\bar{z}$$