Riešenie príkladov z 9. kapitoly

Cvičenie 9.1. Aká je hodnota Boolovej premennej, ktorá je určená podmienkou
(a) $x \cdot \mathbf{1} = \mathbf{0}$ ($\boldsymbol{x}$ = **0**),
(b) $x + x = \mathbf{0}$ (pre $\boldsymbol{x}$ = **0**),
(c) $x \cdot \mathbf{1} = x$ (pre každé $\boldsymbol{x}$),
c(d) $x + \overline{x} = \mathbf{1}$ (pre každé $\boldsymbol{x}$),
(e) $x \cdot \overline{x} = \mathbf{0}$ (pre každé $\boldsymbol{x}$).

Cvičenie 9.2. Zostrojte tabuľku funkčných hodnôt Boolovej funkcie
(a) $f(x,y,z) = \overline{x}y$,

#	x	y	z	f
1	0	0	0	0
2	0	0	1	0
3	0	1	0	1
4	0	1	1	1
5	1	0	0	0
6	1	0	1	0
7	1	1	0	0
8	1	1	1	0

(b) $f(x,y,z) = x + yz$,

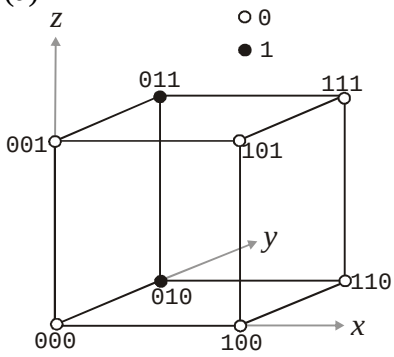
#	x	y	z	f
1	0	0	0	0
2	0	0	1	0
3	0	1	0	0
4	0	1	1	1
5	1	0	0	1
6	1	0	1	1
7	1	1	0	1
8	1	1	1	1

(c) $f(x,y,z) = x\overline{y} + \overline{x}\,\overline{y}\,\overline{z}$,

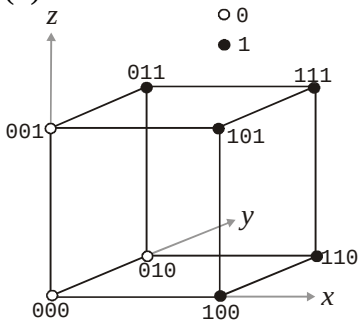
#	x	y	z	$x\overline{y}$	$\overline{x}\,\overline{y}\,\overline{z}$	f
1	0	0	0	0	1	1
2	0	0	1	0	0	0
3	0	1	0	0	0	0
4	0	1	1	0	0	0
5	1	0	0	1	0	1
6	1	0	1	1	0	1
7	1	1	0	0	0	0
8	1	1	1	0	0	0

Cvičenie 9.3. Znázornite Boolove funkcie $f(x,y,z)$ z cvičenia 9.2 na 3-rozmernej kocke tak, že hodnoty 1 (0) budú reprezentované na kocke čiernym (bielym) bodom.

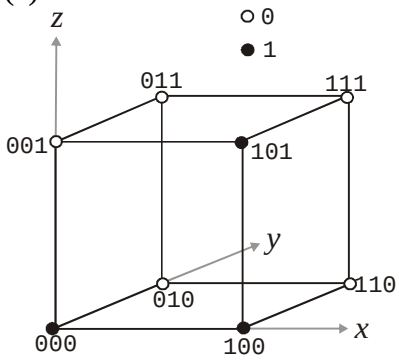
(a)



(b)



(c)



Cvičenie 9.4. Pre ktoré hodnoty *x* a *y* platí $xy = x + y$.

(a) Prípad 0 = 0, potom $x = y = 0$,
(b) Prípad 1 = 1, potom $x = y = 1$.

Cvičenie 9.5. Zostrojte tabuľku všetkých možných binárnych Boolových funkcií z tabuľky 1.8 a identifikujte v nej Boolove binárne operácie pomocou súčinu, súčtu a negácie.

$f_1 = 0$	$f_2 = xy$	$f_3 = x\overline{y}$	$f_4 = x\overline{y} + xy$
$f_5 = \overline{x}y$	$f_6 = \overline{x}y + xy$	$f_7 = \overline{x}y + x\overline{y}$	$f_8 = \overline{x}y + x\overline{y} + xy$
$f_9 = \overline{x}\,\overline{y}$	$f_{10} = \overline{x}\,\overline{y} + xy$	$f_{11} = \overline{x}\,\overline{y} + x\overline{y}$	$f_{12} = \overline{x}\,\overline{y} + x\overline{y} + xy$
$f_{13} = \overline{x}\,\overline{y} + \overline{x}y$	$f_{14} = \overline{x}\,\overline{y} + \overline{x}y + xy$	$f_{15} = \overline{x}\,\overline{y} + \overline{x}y + x\overline{y}$	$f_{16} = 1$

Cvičenie 9.6. Riešte nasledujúce rovnice s exkluzívnou disjunkciou
(a) $x \oplus \mathbf{0} = x$,
(b) $x \oplus \mathbf{1} = \overline{x}$,
(c) $x \oplus x = 0$,
(d) $x \oplus \overline{x} = 1$.

Cvičenie 9.7. Dokážte, že platia rovnosti
(a) $x \oplus y = (x + y)\left(\overline{xy}\right)$,

$$x \oplus y = (x + y)\left(\overline{xy}\right) = (x + y)(\overline{x} + \overline{y}) = \cancel{x\overline{x}} + x\overline{y} + \overline{x}y + \cancel{y\overline{y}}$$

$$= \begin{cases} 0 & (x = y) \\ 1 & (x \neq y) \end{cases}$$

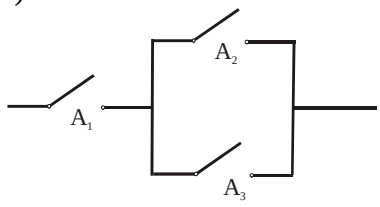
(b) $x \oplus y = \overline{x}y + x\overline{y}$,
Dokáže sa pomocou predošlého riešenia (a).

Cvičenie 9.8. Zostrojte duálne výrazy k týmto Boolovym funkciam
(a) $x + y$, $(\overline{x}\,\overline{y})$
(b) $\overline{x}\,\overline{y}$, $(x + y)$
(c) $xyz + \overline{x}\,\overline{y}\,\overline{z}$, $\left((\overline{x}\,\overline{y}\,\overline{z})(xyz)\right)$.

Cvičenie 9.9. Zostrojte Boolovu funkciu $f(x,y,z)$ vo forme sumy produktov klauzúl k premenným *x*, *y* a *z*, ktorá má hodnotu **1**vtedy a len vtedy, ak
(a) $x = y = \mathbf{0}, z = \mathbf{1}$, $(\overline{x}\,\overline{y}z)$
(b) $x = \mathbf{0}, y = \mathbf{1}, z = \mathbf{0}$, $(\overline{x}\,y\,\overline{z})$
(c) $y = z = \mathbf{1}$, $(xyz + \overline{x}yz)$.

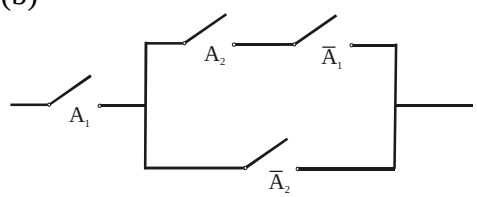
Cvičenie 9.9. Zostrojte spínacie funkcie pre spínacie obvody

(a)



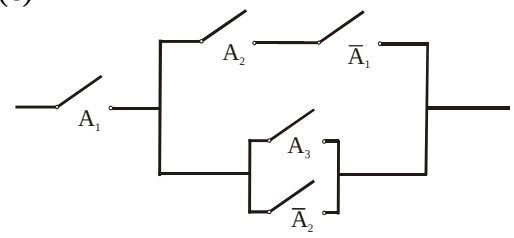
$x_1(x_2 + x_3)$

(b)



$x_1\left((x_2\overline{x}_1) + \overline{x}_2\right)$

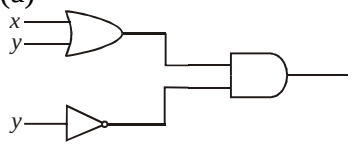
(c)



$x_1\left(x_2\overline{x}_1+\left(x_3+\overline{x}_2\right)\right)$

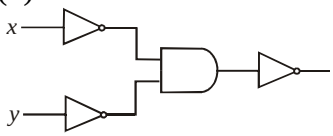
Cvičenie 9.11. Zostrojte tabuľku výstupov logických obvodov

(a)



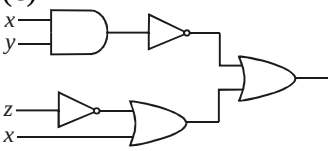
$\left(x\vee y\right)\wedge\neg y$

(b)



$\neg\left(\neg x\wedge\neg y\right)$

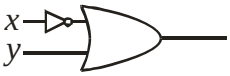
(c)



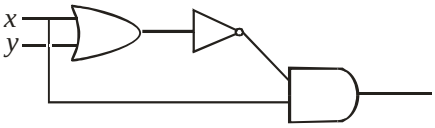
$\left(\neg\left(x\wedge y\right)\right)\vee\left(x\vee\neg z\right)$

Cvičenie 9.12. Zostrojte logické obvody, ktoré simulujú Boolove funkcie

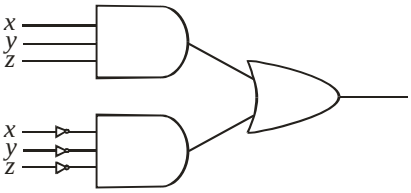
(a) $\overline{x}+y$,



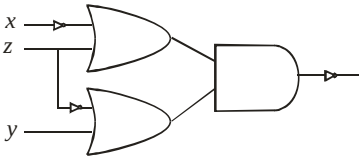
(b) $\left(\overline{x+y}\right)x$,



(c) $xyz+\overline{x}\,\overline{y}\,\overline{x}$,

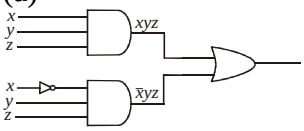


(d) $\overline{(\overline{x}+z)(y+\overline{z})}$.



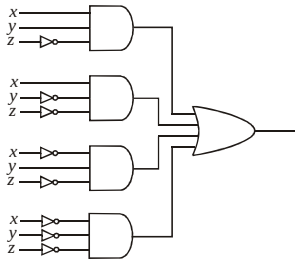
Cvičenie 9.13. Zjednodušte logické obvody

(a)



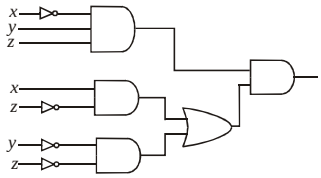
$xyz+\overline{x}yz=(x+\overline{x})yz=yz$

(b)



$xy\overline{z}+x\overline{y}\,\overline{z}+\overline{x}y\overline{z}+\overline{x}\,\overline{y}\,\overline{z}=x(y+\overline{y})\overline{z}+\overline{x}(y+\overline{y})\overline{z}=x\overline{z}+\overline{x}\,\overline{z}=(x+\overline{x})\overline{z}=\overline{z}$

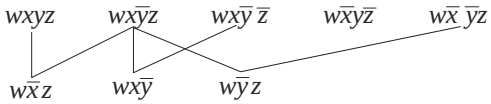
(c)



$(\overline{x}yz)(x\overline{z}+\overline{y}\,\overline{z})=\underbrace{\overline{x}yzx\overline{z}}_{0}+\underbrace{\overline{x}yz\overline{y}\,\overline{z}}_{0}=0$

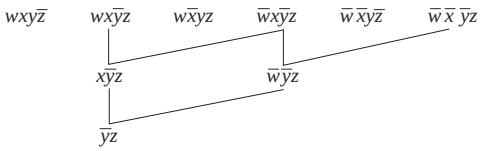
Cvičenie 9.14. Pomocou Quinovej a McCluskeyho metódy nájdite optimálne výrazy k Boolovým funkciám

(a) $wxyz+wx\overline{y}z+wxy\,\overline{z}+w\overline{x}y\overline{z}+w\overline{x}\,\overline{y}z$,



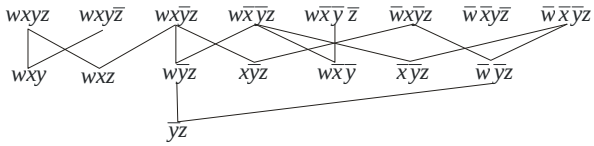
$f=w\overline{x}z+wx\overline{y}+w\overline{y}z+w\overline{x}y\overline{z}$

(b) $wxy\overline{z}+wx\overline{y}z+w\overline{x}yz+\overline{w}xy\overline{z}+\overline{w}\,\overline{x}yz+\overline{w}\,\overline{x}\,\overline{y}z$,



$f=\overline{y}z+wxy\overline{z}+w\overline{x}yz+\overline{w}\,\overline{x}\,\overline{y}z$

(c) $wxyz + wxy\overline{z} + wx\overline{y}z + w\overline{x}\,\overline{y}z + w\overline{x}\,\overline{y}\,\overline{z} + \overline{w}x\overline{y}z + \overline{w}\,\overline{x}y\overline{z} + \overline{w}\,\overline{x}\,\overline{y}z$.



$f = wxy + \overline{y}z + w\overline{x}\,\overline{y} + \overline{w}\overline{x}y\overline{z}$